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=103, As the debenture was discounted, the rate realized on the investment must be greater than 3% as a semi-annual rate. We try 5% and find it too small; we then try 5.0275%, with $(1+x)=1.050275$, and find the first member=102.9337, which is too small; we then try $(1+x)=1.050285$ and obtain the value 102.9531, still too small. By double position we have a corrected value and $(1+x)=1.050309$ and obtain 103.0304, which is now too large. By double positon we have $(1+x)=1.050291$, and obtain 102.9937, again too small. By double position again we have $(1+x)=1.0502941$ and obtain 103.0033, now too large. We now again use double position and obtain, $(1+x)=1.05029269$, this will give a value which will differ by less than 1% of a mill. We now have for a good approximation a semi-annual rate, 5.029269%; or the rate per annum payable semi-annually, 10.058538%.

Also solved by S. A. Corey who got for a result 10.28 per cent.; A. H. Holmes, whose result was 10.07 per cent., and the proposer, whose result was 10.0532 per cent. These contributors used practically the same method of solution. Mr. Spuner also sent in an excellent solution of 345, which reached us too late for credit in the January number.

CALCULUS.

298. Proposed by C. N. SCHMALL, New York City.

Prove, by calculus, that if two regular polygons have equal perimeters, that which has the greater number of sides has the greater area.

Solution by E. L. SHERWOOD, Shady Side Academy, Pittsburg, Penn., and the PROPOSER.

In any regular polygon, let $2p$=the perimeter, n=number of sides.

$\therefore \frac{2p}{n}$=each side, and $\frac{p}{n\tan(\pi/n)}$=the apothem.

$$\frac{p^2}{n\tan(\pi/n)}=\frac{p^2}{n}\cot\frac{\pi}{n}=\text{area}\ldots(1).$$

Put $u=n\tan\frac{\pi}{n}\ldots(2)$; hence the area will be a maximum when u is a minimum.

From (2) we have

$$\frac{du}{dn}=\tan\frac{\pi}{n}-\frac{\pi}{n}\sec^2\frac{\pi}{n}=\tan\phi-\phi\sec^2\phi\ \left(\text{where }\frac{\pi}{n}=\phi\right)=\sin\phi\,\sec\phi-\phi\sec^2\phi$$

$$=\tfrac{1}{2}(2\sin\phi\,\sec\phi-2\phi\sec^2\phi)=\tfrac{1}{2}\sec^2\phi\left(\frac{2\sin\phi}{\sec\phi}-2\phi\right)=\tfrac{1}{2}\sec^2\phi(\sin2\phi-2\phi),$$

$$=\tfrac{1}{2}\sec^2\frac{\pi}{n}\left(\sin\frac{2\pi}{n}-\frac{2\pi}{n}\right).$$

This expression is negative for $n>2$ and therefore decreases as n increases. Hence the area increases as n increases. Q. E. D.

NOTE. Equating the derivative to zero, we have

$$\tan\frac{2\pi}{n}-\frac{2\pi}{n}=0.$$

This equation is true only for very small values of the angle. In fact, it approximates to exactitude as the angle $2\pi/n$ approaches zero. Hence, the area of the polygon opproaches its maximum value (as a limit) as n approaches ∞; it reaches its maximum when $n=\infty$. In that case the polygon becomes a circle, which is, therefore, the greatest of all isoperimetric plane figures. In a similar manner, it may be shown that of all plane figures of given area the circle has the least periphery.

Also solved by J. E. Sanders, V. M. Spunar, J. Scheffer, and S. G. Barton.

299. Proposed by JOSEPH V. COLLINS, Ph. D., State Normal School, Stevens Point, Wisconsin.

A cow is pasturing outside a circular field containing 10 acres. What length of rope will allow her to graze over exactly two acres?

Solution by J. SCHEFFER, A. M., Hagerstown, Md.; J. E. SANDERS, Columbus, Ohio, and S. G. BARTON, Ph. D., Clarkson School of Technology.

It is assumed that the cow is tethered at a point in the circumference of the field.

It is evident that the area of the pasture consists of a semi-circle, of radius equal to the length of the rope and two portions contained between the circular boundary of the field and the involute of the circle.

Let a be the radius of the field, l the length of the rope. The intrinsic equation of the involute of a circle is

$$s=a\frac{\psi^2}{2}.$$

Considering the element of area contained between this involute and its evolute, that is, the circle, and two radii of curvature, we have

$$dA=\rho\frac{ds}{2},\quad \rho=\frac{ds}{d\psi}=a\psi;\quad \therefore s=\frac{\rho^2}{2a},\quad ds=\frac{\rho}{a}d\rho.\quad \therefore dA=\frac{\rho^2}{2a}d\rho.$$

The limits of ρ for area involved are evidently 0 and l.

$$\therefore A=\int_0^l\frac{\rho^2}{2a}d\rho=\frac{l^3}{6a}.\quad \text{Hence area}=2\text{ acres}=\frac{\pi l^2}{2}+\frac{l^3}{3a}.$$